Differentiation

- **Limit definition** of a derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) f(x)}{h}$ •
- Derivatives are used to find the **instantaneous** rate of change of a function.
- Derivative is the slope of the tangent line
- Notation (most commonly used)
 - Leibniz:
 - $\frac{dy}{dx}$ or $\frac{d}{dx}$ for first derivative • $\frac{d^2 y}{dx^2}$ or $\frac{d^2}{dx^2}$ for second derivative
 - $d^n y$ d^n

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$$\frac{d^n y}{dx^n}$$
 or $\frac{d^n}{dx^n}$ for n^{th} derivative

- Lagrange:
 - f'(x) for first derivative
 - f''(x) for second derivative
 - f'''(x) for third derivative
 - $f^4(x)$ for fourth derivative
 - $f^n(x)$ for n^{th} derivative $(\{n \in \mathbb{Z} \mid n > 3\})$
- A function is **differentiable** on an interval if its derivative exists on that interval.
 - Common examples of when a function is not differentiable:
 - Discontinuity
 - Vertical tangent lines
 - Sharp turns
 - If a function is differentiable, then it is continuous.
 - Converse of this statement is NOT true.
- Derivative of a constant is always 0 (constants do not change).
- Common application: motion
 - Let r(t) be a **position function** of an object with respect to time t.
 - Velocity v(t) = r'(t)
 - Acceleration a(t) = r''(t)

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